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Einstein's office at the university [the German University in Prague] overlooked a park with beautiful gardens and shady trees. He noticed that there were only women walking about in the morning and men in the afternoon, and that some walked alone sunk in deep meditation and others gathered in groups and engaged in vehement discussions. On inquiring what this strange garden was, he was told that it was a park belonging to the insane asylum of the province of Bohemia. The people walking in the garden were inmates of this institution, harmless patients who did not have to be confined. When I first went to Prague, Einstein showed me this view, and said playfully: "Those are the madmen who do not occupy themselves with the quantum theory."

—Philipp Frank

ENTROPY

Einstein arrived in Bern in February of 1902. His job at the patent office had not yet materialized so to support himself he decided to give private lessons in physics. He put an ad in the *Anzeiger der Stadt Bern*, the local newspaper, which read

Private lessons in mathematics and physics for students and pupils is given with thoroughness by Albert Einstein, owner of the Swiss polyt.subject teacher diploma, Gerechtigkeitsgasse 32.1st floor. Trial lessons for free.

He got a few takers, one of whom was a Rumanian student of philosophy named Maurice Solovine. Einstein enjoyed talking to Solovine so much that he suggested that they forget about the lessons and simply talk. Not long after they were joined by a young mathematician named Conrad Habicht whom Einstein had already known. The three of them took to meeting regularly in Einstein's flat. A spartan dinner would be served—sausages and the like—there was not much money—and then they would discuss philosophers like Mach or David Hume or even Don Quixote until early hours in the morning. They decided to call themselves as a joke “Akademie Olympia”—the Olympic Academy. On one of Einstein's birthdays Habicht brought caviar which Einstein had never eaten. However, Einstein began a discussion of Galileo's principle of inertia and ate all the caviar without paying any attention to what he had eaten.

By 1905, the Academicians had gone their separate ways. In May of 1905, Habicht received a letter from Einstein which, every time I read it, fills me with the same wonder. He explains that he has written, or is about to write, four papers,

... the first of which I could send off soon, as I am to receive my free copies very shortly. It deals with radiation and the energetic properties of light and is very revolutionary, as you will see provided you send me *your* paper first. The second paper is a determination of the true size of atoms by way of the diffusion and internal friction of

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diluted liquid solutions of neutral substances. [This was Einstein's PhD, thesis which was not published until the following year.] The third proves that, on the assumption of the molecular theory of heat, particles of the order of magnitude of 1/1000 millimeters suspended in liquids must already perform an observable disordered movement, caused by thermal motion. Movements of small inanimate suspended bodies have in fact been observed by physiologists and called by them "Brownian molecular motion." The fourth paper is in the draft stage and is on electrodynamics of moving bodies, applying a modification of the theory of space and time; the purely kinematic part of this paper is certain to interest you.

One barely knows where to begin. Three things strike me at once. First, there is the sheer magnitude of the work involved. Each of these papers has many detailed calculations. He must have been able to perform these with amazing speed and precision. He was, after all, working a full schedule at the patent office and had home responsibilities as well. The second thing that strikes me is his absolutely lucid understanding of what he had done, or was about to do. For example, there is the little phrase "the purely kinematic part of this paper is certain to interest you." "Purely kinematic part" means that part that applies independent of any specified force. It is just here where Einstein separates himself from everyone else. People like Lorentz and Poincaré viewed the problem posed by the Michelson-Morely experiment as a dynamical problem. Lorentz in particular showed how the Michelson-Morely experiment could be "explained" if the electrical forces holding matter together caused a contraction of a moving object like a ruler. As we have seen, these models play no role in Einstein's formulation of relativity. The contraction is a consequence of a modified view of space and time. Finally, there is the characterization of the first paper as being "very revolutionary"—not relativity but radiation. The object of this chapter is to help you to understand why this is so.

Not everyone would do so, but I start the road that finally led to Einstein's 1905 radiation paper, with the invention of the steam engine. The first practical steam engine was invented early in the 18th century by

and again. In Newcomen's engine, cool water was introduced into the piston cylinder after the piston had been raised by the steam. This cooling caused the steam to condense creating a vacuum so that the air pressure above the piston would drive it down, and the cycle could begin again. This form of the steam engine was actually used to drain water from coal pits. But, as an engine, it was very inefficient. Part of the reason was that cooling the cylinder meant that in the next cycle the steam remained partially condensed. I have read that the Newcomen engine was less than one percent efficient. More than 99 percent of the steam power did not do useful work. Enter James Watt.

Watt was born in Greenock, Scotland, in 1736. His father was a maker and supplier of nautical instruments. This is what Watt wanted to do as well. In 1775, he went to London to study this trade, but after a year came back to Scotland. The guild in Glasgow would not accept him. He was fortunate to get a job at the University of Glasgow making and repairing scientific instruments. One of these was a model of a Newcomen engine which Watt was asked to repair. He took the opportunity to study the model and realized how it could be transformed. The problem was that there were two apparently contradictory requirements. On the one hand, the cylinder that housed the piston had to be cooled down so the steam would condense and, on the other hand, it had to be kept hot so that on the next cycle the water would remain vaporized. Watt realized that both requirements could be satisfied if the condenser for the steam was in a vessel that was separate from the cylinder that housed the piston. Figure 4.2 is Watt's first drawing of this arrangement.

By 1765, Watt was writing to people that he had invented the "perfect" steam engine. It is true that the Watt steam engine with its many adumbrations, some invented by Watt, became the basis of the industrial revolution, but was it "perfect?" Indeed, what could such a question possibly mean? The answer was supplied by a French engineer—it is difficult to know what else to call him—named Nicolas Léonard Sadi Carnot.

Carnot, who was born in Paris in 1796, was the son of the polymath Lazare Carnot who worked in a variety of things including mathematics for which he is best known. He also played an important role

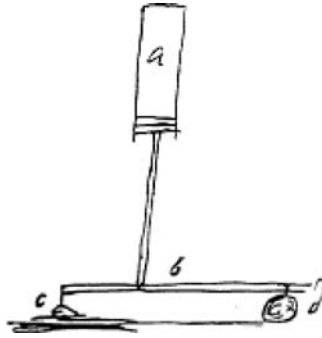


Figure 4.2. In Watson's first drawing of a steam engine, the piston is at the top and the condenser at the bottom.

in Napoleon's government. His son showed considerable brilliance as a student and at sixteen he entered the École Polytechnique in Paris¹ which trained engineers for the military. When Carnot finished his studies he was commissioned as a second lieutenant doing garrison duty in the provinces. In 1819, he was appointed to the army general staff corps in Paris which gave him time to take courses at the Sorbonne and to do his own research. During the next few years he did the work that would become the basis of the later developments 19th-century thermodynamics. In 1824, he published some of it in a 118-page monograph with the long title, *Reflexions sur la puissance motrice du feu et sur les machines propre à développer cette puissance* which I would translate as "Reflections on the motive power of heat and on the machines that are appropriate to making use of this power." During his lifetime—he died of cholera in 1832 at the age of thirty-six—very few people read his monograph. It was only discovered in mid-century along with some unpublished notes that indicate that Carnot was beginning to develop ideas that might well have led him to the kinetic theory of heat. As it was, and as I will now explain, his monograph was written with a totally wrong theory of heat in mind, and it did not matter. What was this wrong theory of heat?

¹I spent the year 1959–1960 at the École Polytechnic which was still located in its ancient buildings in the Latin Quarter in Paris. The students were all in uniform. My patron, the late Louis Michel, a brilliant theoretical physicist, was a graduate. He too served for a time as a military engineer.

Carnot believed in the “caloric” theory of heat. He was in good company. For example, Benjamin Franklin measured heat flow down rods made of various materials to see how long it took to melt wax. He came to the conclusion that heat was a weightless fluid—caloric—which was conserved in all processes. He had correctly postulated that electricity flowed in currents. This time he missed. It was well known that friction produced heat. This was explained by arguing that when the objects rubbed against each other, caloric was transferred. There was, however, some evidence that argued against this. An American named William Thompson ended up in Bavaria where he was knighted. He became Count Rumford—Rumford being an old name for Concord, New Hampshire. While in Bavaria, Rumford occupied himself with, among other things, cannons. He was impressed by the amount of heat that is generated in cannon barrels by the explosion of gun powder and concluded that there could not be enough caloric stored to account for this. He made the same observation when he drilled out metal to make the barrels of cannons. Indeed, using the heat generated in such an operation he made water boil. He decided that something like a molecular theory of heat must be right and published this in 1798, to no effect. It took until the middle of the next century before the caloric theory of heat was finally disposed of. As it turned out, Carnot’s belief in an incorrect theory of heat had no relevance to what he did.

Carnot was interested in what would constitute a perfect steam engine. This was a conceptual engine, but he thought that it might play some role in building a real steam engine. The key idea in Carnot’s thinking about engines was the distinction he made between reversible and irreversible processes. In real life the processes that we encounter are irreversible. As the nursery rhyme has it, “All the king’s horses/And all the king’s men/Couldn’t put Humpty together again.” Reversible processes are, like Einstein’s trains or elevators, thought experiments that illuminate laws of physics. In Carnot’s “engine” there was a cylinder and a piston. The piston is assumed to work fictionlessly. In the cylinder was a gas—steam if you like—that was assumed to obey Boyle’s law—a “perfect” gas. It turned out that Carnot’s results did not depend on the specific

choice of the working gas. In addition there were two heat reservoirs maintained at different temperatures. Heat—caloric in Carnot’s case—can be added or subtracted from the reservoirs as needed. Two kinds of processes were allowed. In the first process heat is absorbed from the reservoir, which is at the same temperature as the ambient temperature of the gas, in such a way that the temperature of the gas remains the same while the pressure and volume adjust. This absorbed heat raises the piston. Such a process is called “isothermal” and, if carried out carefully, is reversible. The second kind of process is called “adiabatic.” We now decouple the cylinder from the heat source and let the piston expand very slowly. This will reduce the temperature to that of the colder reservoir. If we do this, again very carefully, then it is reversible. Now we can put these two types of reversible processes together in a cycle which is a conceptual model for a reversible engine. It does not matter where we start the cycle or even in which direction we run it but conventionally it works like this (Figure 4.3).

- Step 1. This is an isothermal expansion that maintains the temperature but during which the gas expands. Heat is absorbed.
- Step 2. We now make an adiabatic expansion which reduces the temperature to that of the cold reservoir.
- Step 3. We place the cylinder in contact with the cold reservoir and draw off the heat absorbed in Step 1.
- Step 4. We make an adiabatic compression of the gas so as to return it to its initial condition.

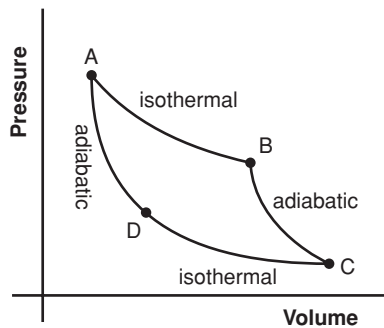


Figure 4.3. The four steps of Carnot's cycle.

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These four steps are what is called the Carnot Cycle. Each of them is reversible. This makes it an efficient engine but what Carnot went on to show is that no engine can be more efficient than this.

To do this Carnot invoked a mantra that became a centerpiece for all the 19th-century development of thermodynamics; there is no such thing as perpetual motion. It is a curious statement if one thinks about it. How would you prove it? It is true that all the motions we have studied eventually come to a stop. But “eventually” is not “perpetual.” Nonetheless, this doctrine was, and is, taken as a law of physics. What Carnot showed is that if there were a machine more efficient than his cycle it could be hooked to his cycle, run backward, and operate perpetually. Thus the Carnot cycle is the idealized limit of what can be achieved in making an efficient engine.

As I have mentioned, Carnot died young and his work was for some time not much appreciated. In midcentury he was rediscovered by a German physicist named Rudolph Clausius, who was born in what was then Prussia in 1822. It was in 1850, that Clausius wrote the paper that laid the foundations for thermodynamics. Clausius started from two principles. The first one was known as the mechanical equivalence of heat, something that we would call the conservation of energy. This had been established with some certainty by the experiments done by the British physicist James Joule—pronounced “jowell”—who was the son of a prosperous Manchester brewer. Joule had become convinced that the caloric theory was defective. It did not seem to account for the results of his experiments. In particular, he churned water with a paddle and found that this produced a measurable quantity of heat in an insulated can. This was difficult to account for by a transfer of caloric. In our terms, mechanical energy was being transformed into an equivalent amount of heat energy. This was one of Clausius’s starting axioms.

His second principle was the proposition that you cannot transfer heat from a cold body to a hotter one without supplying some sort of energy. At first this seems crazy. How do you cool down soup? If one thinks about it, the air is cooler than the soup so we are transferring heat from a hot body to a cooler one, which is allowed. You can cool things

in a refrigerator providing that you pay the electricity bill to run the motor. After he published his 1850 paper, Clausius put his second law in a different form. He introduced a concept that he named “entropy.” He informs us that he took it from the Greek *ητροπη*—meaning a transformation or a turning. In classical thermodynamics the concept of entropy is somewhat slippery because what is well defined is the difference in entropy of two thermodynamic states—states that are characterized by quantities such as heat content and temperature.² In particular, if we have two identical heat reservoirs with heat content, Q , but at different temperatures, T and T^* then Clausius said that the difference in entropy between these states ΔS is given by

$$\Delta S = Q \left(\frac{1}{T} - \frac{1}{T^*} \right).$$

You cannot **prove** that this is the difference of entropy. It is how Clausius defined entropy. But how do you make use of this definition? What Clausius realized was that if you demanded that in all processes in a closed system entropy never **decreased** you could subsume in one law—this is the “second law of thermodynamics,” the first being energy conservation—all the strictures on perpetual motion and the impossibility of transferring heat from a cold to a hotter reservoir without supplying energy from the outside and the rest.

To see how this works in a couple of examples let us begin with Carnot and his cycle. Carnot, as a consequence of his use of the caloric theory of heat in which heat is conserved in every transaction, would say

²In thermodynamics there is a third law that was formulated by the chemist Walther Nernst at the beginning of the 20th century. It says that as the temperature falls to absolute zero the entropy also falls to zero. Thus, given the third law, it is possible to define the absolute entropy of a system. The problem is that for, say, classical perfect gasses, the third law fails. For a classical perfect gas the entropy goes as the logarithm of the temperature and thus the third law is violated. Thus classically only the relative entropy is defined. I am grateful to Freeman Dyson for discussions of this.

that, at the end of his cycle, the reservoir would have exactly the heat content it started with. Furthermore, you have the same temperature initially and finally. Thus $\Delta S = 0$. In other words, if the cycle consists of reversible processes, then the change in entropy is zero. This is the limiting case of the second law. Now suppose that you try to make a transfer from a colder to a hotter reservoir without supplying additional heat energy. In this case $Q_f = Q_i = Q$, since no heat has been transferred. Thus $\Delta S = Q(\frac{1}{T_f} - \frac{1}{T_i})$. Therefore, if the final temperature is larger than the initial one, the change the change in entropy is negative which violates Clausius's second law. Therefore the second law prohibits such unassisted transfers. Clausius had his own mantra. It read *Die Energie der Welt ist constant. Die Entropie der Welt strebt einem Maximum zu*. "The energy of the universe is constant. The entropy of the universe tends to a maximum." By the time Clausius died in 1888, the first and second laws of thermodynamics were accepted parts of physics.

By this time, you may be saying, while this is all very nice, what does it have to do with the quantum? Patience.

Black bodies

On thermodynamic grounds Kirchhoff had concluded that the energy density and the spectral composition of radiation in a *Hohlraum*, surrounded by impenetrable walls of the temperature T , would be independent of the nature of the walls.

— Albert Einstein

We are now going to build a bridge between this 19th-century thermodynamics and the quantum. New actors will come on stage. The first of these is the German physicist Gustav Robert Kirchhoff who was born in 1824. He came from an socially conscious intellectual family who felt that being a university professor filled a civil obligation. Kirchhoff studied at the University of Königsberg and, while a student, made his first important discovery. His professor, Franz Neumann, was interested in electric circuits so Kirchhoff became interested in them too. He developed a set of relations that tell us how currents flow—"Kirchhoff's laws"—which we teach to undergraduates to this day. Kirchhoff first acquired a post

in Berlin, then in 1850, in Breslau, where he met the chemist Robert Bunsen, who was somewhat older and was permanently attached to the University of Heidelberg. In 1854, Bunsen invited Kirchhoff to join him there. Bunsen was the inventor of the eponymous burner so beloved of chemists. He was in the process of using the burner to heat various elements to incandescence. Kirchhoff proposed that they use a spectrometer to measure the light spectrum, unique to each element, when it becomes incandescent. Over the next few years they discovered cesium and rubidium this way. They also used the spectrometer to extend work done earlier by Joseph Fraunhofer to study light coming from the Sun. Fraunhofer had noticed that, at some wavelengths, instead of seeing bright lines, there were dark ones. Bunsen and Kirchhoff observed that if sodium vapor was put in front of the spectroscopy these lines became even darker. Indeed, if they studied incandescent sodium through sodium vapor they saw the same effect. The conclusion they drew was that sodium could both emit and absorb light at the same wavelength. They had no explanation for this, nor was any forthcoming until the advent of the quantum theory.

It was this work that led Kirchhoff to do the analysis of radiation that is most relevant to us. This analysis is philosophically somewhat akin to what Carnot did for engines. It does not deal with something that can be realized exactly in nature—although one must say that the background radiation left over from the Big Bang comes pretty close. It is an idealized situation that can teach us much. In his analysis he imagined a container whose walls can be heated up. It will turn out from Kirchhoff's analysis that it does not matter what the container is made of. Of course you do not want it to melt. Nor does its size and shape matter. I will now be a little anachronistic in describing what happens—a little, because by the end of the century this was the picture that had been adopted. Because they are heated, the electrons in the atoms of the material that composes the container walls will begin to oscillate. These oscillating electrons will radiate, and this radiation will enter the container, where some of it will be reabsorbed by the walls, and then reemitted. In due course, a

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situation of thermal equilibrium will be established in which the emission and absorption will just balance. At equilibrium the radiation will be characterized by some temperature T . When this happens there will be a distribution of different wave lengths of the radiation in the cavity. Some wave lengths will be favored and will be present with greater intensity than the less favored wave lengths. There is no reason to think that any wave length is excluded—from the longest to the shortest. It is just that they will have different intensities.

Now, we can imagine measuring these intensities and plotting the result in a graph. This graph will be representable by curve that, if we are lucky, will have a simple functional form. The question that Kirchhoff asked, and answered, is on what does this function depend? At first sight, we can think of a lot of things. It might depend on the size and shape of the cavity, or on what material it's made of. We know it will depend on the wave length, and surely it will depend on the temperature. As you heat things up, the dominant light you see changes color. That's what you mean, for example, when you say something is "white hot." What Kirchhoff showed is that, in fact, wave length and temperature are **all** this function depends on. Here is where the thermodynamics comes in.

Let us assume the contrary. Let us assume for the sake of argument that two containers made of different materials, with different sizes and shapes, have, at the same temperature different equilibrium radiation distributions. In one container some wave lengths may be favored and in the other, the same wave lengths will have, relatively speaking, less intensity. Now what we can do is to build a contraption that puts these containers together with a little door connecting them that we can open and close. With the door open, radiation will pass from one container to the other. Through the open door, radiation with the favored wave lengths will enter the second container to fill out its distribution. This is analogous to any kind of diffusion in which the diffusing material goes from places of greater to lesser density. But, in this case, it has done so between two regions at the same temperature with no external energy being involved. Now, if we close the little door, the two regions will again

come to equilibrium, but at different temperatures, since the transport of the radiation has also transported energy. We have as a result a hotter and colder reservoir and can again open the little door. Heat will be transported back across from the hotter to the cooler reservoir. It is clear that we are starting to generate a perpetual motion machine, so the premise must be wrong. The two distributions must be the same. We can also draw a similar conclusion if we stick to one container and assume that the radiation distribution varies from place to place within it. The equilibrium distribution takes the same form no matter where we measure it in the container. Thus Kirchhoff drew the remarkable conclusion that this equilibrium radiation distribution was a universal function that depended only on wave length and temperature, and nothing else. But this was not all.

To appreciate the next step, let us examine the emission and absorption processes a little more closely. The emission will be governed by a relative probability function that tells us how likely the electrons in the wall of the container are to emit radiation with some given set of wave lengths. The walls of the container are at some temperature, T , which will be implicit in the discussion that follows. Let us call this function; “ $emission(\lambda)$,” where λ is the wave length. There will also be a corresponding absorption function. Here, there is a little subtlety. You can’t absorb radiation that is not present. What is present is determined by Kirchhoff’s universal function which I will call “ $universal(\lambda)$.” Thus the absorption is given by the product; $absorption(\lambda) \times universal(\lambda)$. At equilibrium, emission and absorption balance so that $emission(\lambda) = absorption(\lambda) \times universal(\lambda)$ or $\frac{emission(\lambda)}{absorption(\lambda)} = universal(\lambda)$. Now comes the punch line. Kirchhoff imagined an object, which in 1862 he named a “black body,” that is, a perfect absorber. It absorbs all wave lengths with equal avidity. In terms of our definitions $absorption(\lambda) = 1$. So that $emission(\lambda) = universal(\lambda)$. In nature there are no perfect black bodies, although I will shortly give you two very good approximations. What the equation says is that, for a black body which is in equilibrium with its radiation, we can measure $universal(\lambda)$ by measuring the emission

spectrum. This is something we might be able to get our hands on if we can identify something that acts at least approximately like a black body. We should be able to study the emission coming from it.

Here are two examples of entities that act like black bodies. The first is just a container with a small hole in it. Any radiation that falls on the hole will be absorbed by it and will have great difficulty escaping. Now, suppose you heat up the container. An equilibrium distribution will develop. A bit of this radiation will be emitted by the hole. By Kirchhoff's result, this radiation will be characterized by *universal*(λ). Thus, by measuring the radiation coming from the hole, we can measure the black-body spectrum of radiation. This, in essence, is how it was determined experimentally. The second example is much more exotic. It involves the early universe. Until something like 400,000 years after the Big Bang, matter in the early universe was in the form of a plasma. The electrically charged particles that constitute the plasma were primarily electrons and protons. There were also neutrinos and radiation left over from the original explosion. Radiation scatters from these electrons and an equilibrium is established. If you like, in this respect, the whole universe was a black body. Until the temperature fell to less than about $3,000^\circ$ —which happens at something like 400,000 years—the hot radiation will not let neutral hydrogen atoms form. They would get ripped apart. But at a temperature of about $3,000^\circ$, the radiation is too cool to do this. Electrons and protons can now form electrically neutral hydrogen atoms and the radiation is free to expand with the universe. It has no charged particles to scatter from. Since nothing interferes, whatever black body distribution it had at the epoch of electron-proton combination, will be present now for us to observe, but, of course, at a vastly lower temperature. Indeed, this distribution has been observed with great precision. The black-body curve which arises from the experiments—I will show one later—is so good that, when you first see it, you think it must have been computer generated. It corresponds to a present temperature of about 2.73° above absolute zero, the average temperature of the present universe. With these examples in mind, we can now return to our historical narrative.

In 1879, the Austrian physicist Josef Stefan, who was Ludwig Boltzmann's teacher,³ took an important step. We have not discussed the energy that a black body emits. To make the discussion not dependent on the size or shape of the hole in the container, one discusses the energy emitted per square centimeter of the surface of the hole, per second. One would guess that this "flux" would depend on the temperature of the container. The hotter it is, one would guess, the greater would be the energy flux. It is unlikely, however, that one would guess just how radical this dependence is. Stefan found that to fit the limited experimental data available, he had to assume that the flux varied as the **fourth** power of the temperature— T^4 . If you, for example, double the temperature the flux will go up by factor of sixteen. This means that by doubling the temperature, a black body will radiate its energy away sixteen times as fast. In 1884, using methods of kinetic theory, Boltzmann was able to derive this result from first principles. But Stefan's empirical result was an important landmark.

The penultimate figure in this bridge building section is the German physicist Wilhelm Wien, who was born in East Prussia in 1864. When he was a young man he had difficulty deciding whether he wanted to be a physicist or a farmer. Einstein used to say that the best profession for a physicist would be as a lighthouse keeper, because you could just sit there all day thinking about physics. Of course Einstein became a university professor in some of the best universities in the world. Wien did not want to be a farmer to do physics. He wanted to be a farmer, period. By 1890, his parents were forced to sell the family farm, and Wien became a physicist for good. He took positions first at the University of Berlin, and ultimately at the University of Munich, where he remained until his death in 1928. From our point of view he did two essential things. One of them followed from a thermodynamic argument that goes beyond what we can present here, and the other from an inspired guess. Fortunately the consequences of these arguments are easy to state. Both have to do with

³My teacher, Philipp Frank, was Boltzmann's student. The chain gives me an odd feeling.

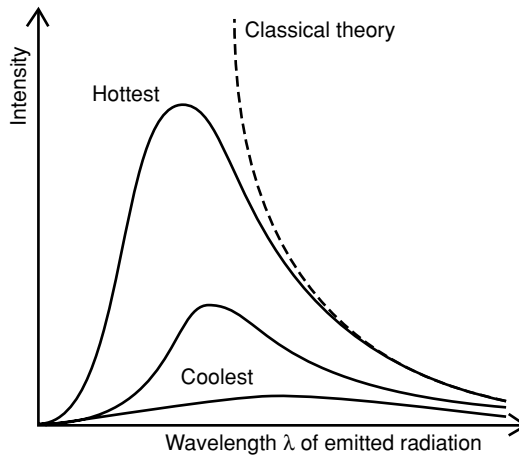


Figure 4.4. The short wave length end of the spectrum fits the Wien distribution.

determining $universal(\lambda)$. To appreciate the first one we have to recall that $universal(\lambda)$ is also a function of the temperature. It is more correct to write it as $universal(\lambda, T)$, a function of two variables. On its face this makes it much more difficult to find the function. The possibilities are now in two dimensions, which vastly widens the search. But, using thermodynamics, Wien proved a remarkable result. He showed that $universal(\lambda, T)$ takes the form $\frac{1}{\lambda^5} \times Universal(\lambda T)$. This means that the unknown function, which I have called “Universal” to distinguish it from “universal,” is only a function of one variable, λT . This makes the problem of finding it even more tantalizing. In fact, using inspired guessing, Wien wrote down a form that turned out to be correct for part of the spectrum. Indeed, as we shall see, this was the very part that Einstein analyzed. I have a graph above that shows the full experimental spectrum at various temperatures. We will discuss the full graph soon. But, for the moment, I just want you to focus on the short wave length end (Figure 4.4). The long wave length end is labeled, for reasons I will also explain, “classical theory.” At the short wave length end, you see from the graph that the spectrum falls rapidly to zero. This end of the spectrum is referred to as “ultraviolet” — beyond violet. Indeed, the fall at the ultraviolet end is exponential. This is precisely is what Wien proposed. His argument was pretty shaky, but his exponential form did seem to agree with whatever experimental data

was available.⁴ The year is now approaching 1890, and we have made some progress. But it is time introduce the last actor in this part of the drama, Max Planck.

THE RELUCTANT RADICAL

One should proceed as conservatively as possible in introducing the quantum of action into the theory, making only those changes in the existing theory that have proved to be absolutely necessary.

—Max Planck

In describing Planck the word “conservative” comes immediately to mind. It was almost genetic. Planck, who was born in 1858, in Kiel, where his father was a professor of civil law, came from a long line of lawyers and clergymen. He, and they, were deeply immersed in German culture. Planck was culturally, politically, and scientifically conservative. He and Einstein were antipodes. What they had in common was that they lived in troubled times. In World War I, a group of 93 very prominent German intellectuals, including Planck, signed a manifesto entitled *Appeal to the Cultured World*. It was a justification of German militarism. It indicted the French and British for “having allied themselves with Russians and Serbs, and presenting to the world the shameful spectacle of Mongols and Negroes being driven against the white race.” Needless to say, Einstein did not sign this document. Planck’s oldest son Karl was killed in the war. After Hitler came to power, Planck, who had won the Nobel Prize in 1918, and was certainly one of the most recognized scientists in Germany, went to see him to try to convince him to temper his anti-Semitic laws against Jewish scientists in the universities. At the mention of Einstein,

⁴In formulae what Wien proposed was that $universal(\lambda) = a \frac{1}{\lambda^5} \exp(-\frac{b}{\lambda T})$ where a and b are constants that have to be adjusted to fit the data. He got this expression by using as an analogy Maxwell’s formula for the way the energy is distributed among the molecules in a heated gas. It never occurred to Wien that if this analogy worked it might mean that black body radiation was behaving like a gas of particles.

Hitler went into a tantrum of rage. Planck retreated and spent the rest of the Nazi period attempting to salvage what he could of German science. In July of 1944, his younger son Erwin took part in an unsuccessful attempt to assassinate Hitler. He was arrested and died a terrible death at the hands of the Gestapo. Planck survived the war, dying in 1947.

As a child he was clearly gifted. He excelled in both music and mathematics. When he graduated from the *Gymnasium* in 1874, he briefly entertained the idea of becoming a pianist. However, he entered the University of Munich to study physics. He spent the year 1877–1878 in Berlin, where he attended lectures of Kirchhoff. Kirchhoff had moved to Berlin, and given up his experimental work, because an accident had left him crippled. He spent the later half of his life on crutches or in a wheel chair. But he was still lecturing. Planck was somewhat less than enthusiastic. He later wrote,

I must acknowledge that I gained little from the lectures. . . . Therefore I could only still my need for continuing scientific education by reading works that interested me, and those naturally were ones relating to the energy principle [the conservation of energy]. In this way I came upon the papers of Rudolph Clausius, whose clarity of expression and thought made a powerful impression. With growing enthusiasm I worked my way deeply into them. What I particularly admired was the exact formulation of the two laws of thermodynamics. . . .

This is, of course a reference to the first law—the conservation of energy—and the second law—that entropy never decreases. As Planck then viewed these two laws they were on the same footing. Both were absolute statements about nature, which appealed to Planck’s scientific conservatism. Planck was also a Machian anti-atomist. He soon found himself in a conflict with Boltzmann.

Boltzmann was a confirmed atomist who believed that the laws of thermodynamics reflected the statistical behavior of molecules. Heat, for example, was simply a manifestation of the disordered motion of molecules. But, when it came to entropy there was a dilemma. It had been posed to Boltzmann by his senior colleague and teacher Loschmidt.

Molecular collisions were reversible. To any collision there was a reverse collision with the initial and final momenta interchanged. If you filmed the collisions you could not tell whether the film was running forward or backward. How then did what appeared to be an absolute law of irreversibility—that entropy never decreased—arise? Boltzmann's solution was to argue that, unlike the first law which says that energy is **always** conserved, the second law says only that entropy **probably** never decreases. It is astronomically unlikely that “humpty” will spontaneously be put together again, but it is not impossible. Boltzmann spent a number of years showing explicitly how systems evolve towards more and more probable configurations, which is what he meant by an increase in entropy, always allowing for the possibility that there can be fluctuations. This is what, according to Boltzmann, the second law means.⁵ This interpretation was, in his early years, an anathema, to Planck. In 1882, he wrote a paper in which he concluded,

Consistently developed, the second law of the mechanical theory of heat is incompatible with the assumption of finite atoms. It can therefore be foreseen that the further development of the theory will lead to a battle of these two hypotheses in which one of them will perish. An attempt to predict the conflict's outcome with precision at this time would be premature. Nevertheless, a variety of present signs seems to me to indicate that atomic theory, despite its great success, will ultimately have to be abandoned in favor of continuous matter.

By the turn of the century Planck had changed his mind.

It was almost inevitable that Planck would become involved with black-body radiation. In 1889, after Kirchhoff's death, he succeeded him in Berlin. Berlin was the world center for the study of black-body radiation. Wien was there when, in 1896, he proposed his form of the

⁵Boltzmann expressed this in a formula. If, S , is the entropy of a state, and, P , is the probability of that state's occurrence then $S = -k \log(P)$ where k is a constant—Boltzmann's constant. Boltzmann was so pleased with this expression that he had a version of it engraved on his tombstone.

distribution. In addition, there were two very powerful experimental teams measuring the spectrum of the radiation, that of Otto Lummer and Ernst Pringheim and that of Heinrich Rubens and Fernand Kurlbaum. What experimental evidence there was when Planck began considering the problem, agreed with the Wien law. For Planck, the problem of black-body radiation was therefore how to derive the Wien law from first principles. For the next decade he produced various arguments each one of which he was sure was definitive, and each one of which was flawed. Whatever else, it gave him a mastery of the techniques. It also gave him a constant. This is one of the strangest aspects of the story. The Wien formula had two arbitrary constants that could be adjusted to fit the data. One of them had the peculiar dimensions of energy \times time. Planck argued that it must be a fundamental constant of nature since the black-body spectrum was universal. He realized that this constant enabled him to complete a set of units that had universal meaning. What I mean is this. Take, for example, the usual unit of length, the meter. It was originally defined as a ten millionth of the length of the meridian through Paris from a pole to the equator. How would you explain this to an extraterrestrial? But you could explain the charge of the electron, or the strength of the gravitational force. These are universal units. There are no references to Paris. You could also explain Planck's new constant, assuming the extraterrestrial could construct a black body. As Planck wrote, his units would be "independent of particular bodies or substances, would necessarily retain their significance for all times and all cultures, including extraterrestrial and non-human ones." Planck, with his conservative bent, and love of the universal, was enthralled by this new set of units. We are enthralled by them too. To take an example, if we call Planck's constant h , The gravitational constant, G , and the speed of light c , then the Planck length l_p is given by $l_p = \sqrt{\frac{Gh}{c^3}} \simeq 4 \times 10^{-33} \text{ cm}$. As we will now see, it was his new constant that heralded the onset of the quantum. What is strange about this was that he discovered this unit before he discovered that there was a quantum.

By the fall of 1900, it was becoming clear that the wheels were coming off. Both of the experimental groups had been able to extend their

measurements to longer wave lengths and these results did not agree with Wien. Planck apparently learned about the new results on the afternoon of October 7, when Rubens and his wife came for a visit. For long wave-lengths the spectral function appeared to be going to zero proportional to the wave length, i.e., $universal(\lambda T) \approx \lambda T$. Thus at short wave lengths the function went to zero exponentially, while at long wave lengths it went to zero with λ . Planck's problem was to find a distribution that did both of these things. By that evening, Planck had come up with one, largely inspired guess work, based on years of immersion in the problem.⁶ The new distribution had two virtues. It agreed with Wien for small wave lengths and with the new results at longer wave lengths. Indeed, it agreed with experiment at all the wave lengths that had been measured. It still does. Figure 4.4 showed Planck's distribution for three temperatures. The graphs were prepared using Planck's formula—and not the experiments. But Figure 4.5 is the measured black-body spectrum left over from the radiation produced by the Big Bang, with the present temperature of about 2.73 degrees above absolute zero. You cannot distinguish the experimental from the theoretical curve.

Now Planck had a new problem; how to derive his distribution.

In accounting for what Planck did in 1900, and 1901, there is some difficulty. Part of the problem is that Planck's papers, taken on their own terms, are opaque. But the real problem is that Planck was trying to do something that was impossible. He was trying to derive his distribution from classical physics. He spent a decade trying. Late in life he wrote a scientific autobiography in which he says,

My futile attempts to fit the elementary quantum of action into the classical theory continued for a number of years, and they cost me a great deal of effort. Many of my colleagues saw in this something bordering on tragedy. But I feel differently about it.

⁶In the notation of footnote 4 what Planck did was to replace $e^{-\frac{b}{\lambda T}}$ by $\frac{1}{e^{\frac{b}{\lambda T}} - 1}$. As my teacher Philipp Frank used to say, those of you who know a little of mathematics can show that this distribution morphs into that of Wien when the wave length becomes small.

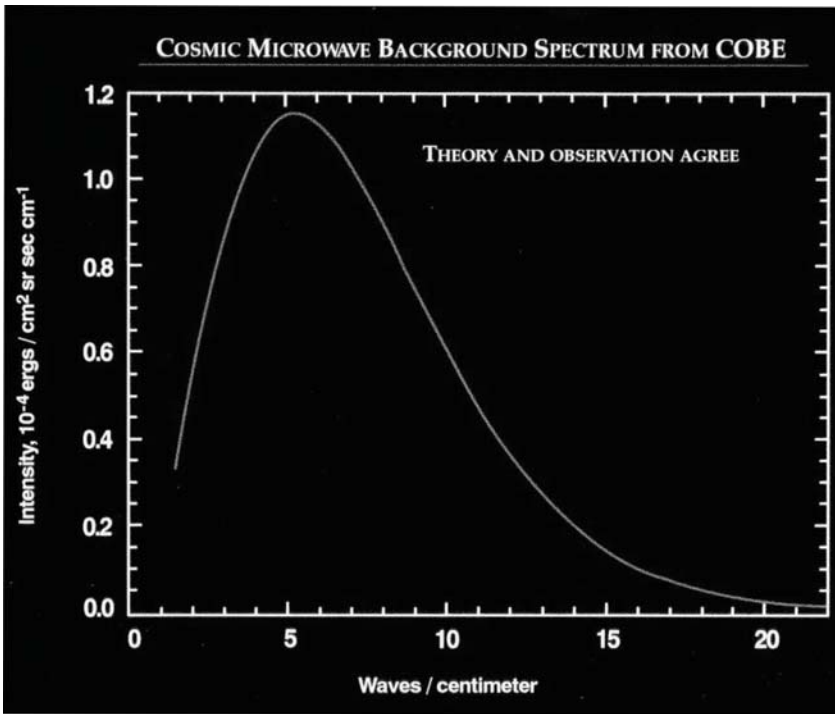


Figure 4.5. The experimental black-body spectrum, for the cosmic microwaves cannot be distinguished from the theoretical.

For the thorough enlightenment I thus received was all the more valuable. I now knew for a fact that the elementary quantum of action played a far more significant part in physics than I had originally been inclined to suspect.

What there is no disagreement about was how he went about the problem. It was an extension of what he had done to try to derive the Wien distribution.

In the first place, there was the model. The electrons in the atoms that make up the walls of the black body container are set into oscillation when the walls are heated. These oscillating electrons—which Planck referred to as “resonators”—emit—and absorb—radiation. An equilibrium will be reached in which emission and absorption balance. At this equilibrium the oscillators will have an entropy. Planck showed that if he knew the functional form of the entropy he could use the laws of

thermodynamics to work his way back to the distribution of radiation. This is one of the things he came to understand in the decade he worked on this problem before 1900. Thus it came down to finding this entropy. By 1900, Planck had accepted Boltzmann's idea that to find the entropy you had to find the probability of various configurations of the distribution of energy among the oscillators. As the system evolves towards equilibrium the system evolves towards more probable states until it reaches the state of maximum probability at equilibrium. So the problem came down to finding the most probable distribution of the energies among the oscillators. This would be the black body distribution. It was precisely at this point that the quantum came in.

Finding probabilities is a counting problem. You count the number of faces on a die — six — and you conclude that the probability of throwing a one is one-sixth. Planck was forced to reduce his probability question to a counting problem. It is here that the historians disagree as to how he went about it. I will tell you the story that I first learned from my teacher Philipp Frank. It is the one that we tell our students. Put anachronistically, according to this version, Planck “quantized” the oscillators. Here is what this means. A classical oscillator can absorb and emit radiation of any energy from zero to infinity. A quantized oscillator has restrictions. Suppose we call the basic energy unit E , then the quantized oscillator can only emit and absorb radiation that has the values E , $2E$, $3E$, $4E$, and so on. These units of energy Planck called “quanta.” Professor Frank had a homely analogy. He said that it is like buying and selling beer in pints and quarts. I suppose the classical oscillator was like buying and selling beer on tap, although I do not recall Professor Frank as having made that analogy. In terms of these quantized energies Planck could carry out his counting to compute the probabilities. There was some hanky-panky in his counting procedure as well. The full meaning of this was not understood for many years.⁷ Be that as it may, with these rather

⁷Not to be too laconic about this, when Boltzmann did his counting he assumed that his molecules were “distinguishable.” Metaphorically speaking they came

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odd assumptions Planck was able to derive his distribution provided that he set $E = h\nu$. Here, ν is the frequency of the radiation and h is the constant with the strange dimensions that Planck had introduced in his version of the Wien distribution. Using his 1901 data, Planck found $h = 6.55 \times 10^{-27}$ erg. seconds, while the modern value is 6.63×10^{-27} erg. seconds. The “erg” being a standard unit of energy. Recall that there was a second constant to be determined in the Wien distribution. This one can be related to Loschmidt’s number; the number of molecules in a cubic centimeter of a standard gas. From the data Planck found 2.76×10^{19} molecules per cubic centimeter to be compared to the modern value of 2.69×10^{19} molecules per cubic centimeter. It was at the time the most accurate determination that had been made.

I have given you the standard version of how Planck made his discovery, but historians who have studied his papers and correspondence carefully do not think that it is correct. They may well be right. They point out that nowhere in his early papers does Planck say anything about quantizing individual oscillators. What he does do is to take a group of oscillators and stipulate that their total energy is an integer multiple of a basic quantum unit. This was a device that Boltzmann used when he computed probabilities. But then, at the end of his calculations, Boltzmann allowed the energies to become continuous again. Planck could not do this because it gave him the wrong answer. If he did it he was lead back to the Wien distribution. Planck was, at least temporarily he thought, stuck with the quantum. Because of the way he did it, he did not think at first that he had done anything radical. He was sure that if he kept working at it he would find a way of deriving his distribution from classical physics. The real revolutionary is now coming on stage.

in different colors. Planck, with no justification, assumed that his quanta were “indistinguishable.” This completely changes the counting problem. If he had not made this assumption he would have ended up with the wrong distribution. Nearly three decades later this was understood after the creation of the quantum theory.

☞ PHOTONS

It appears to me, in fact, that the observations on “black-body radiation,” photoluminescence, the generating of cathode rays with ultraviolet radiation, and other groups of phenomena related to the generation and transformation of light can be understood better on the assumption that the energy in light is distributed discontinuously in space.”

—Albert Einstein

Of Einstein’s four “Miracle Year” papers, his paper on black-body radiation is to me the most miraculous. This is to take nothing away from the other papers, but here he is truly groping in the dark. Here he is at age twenty-six, with no academic job, barely known in the physics community, and no other professional physicists to talk to, not even an adequate library, about to take down the entire edifice of classical physics. I have often thought that if Einstein and Maxwell had been able to sit down and have a chat, Maxwell would have understood relativity in an hour. With Newton you would have had to explain too much. The Brownian movement paper was certainly a fine paper but, after all, it was done almost as well by someone else — Smoluchowski. No one else could have written the black-body paper and no one else believed the results. When Planck, and three colleagues, proposed Einstein for membership in the Prussian Academy they wrote the following:

In sum, one can say that there is hardly one among the great problems in which modern physics is so rich to which Einstein has not made a remarkable contribution. That he may sometimes have missed the target in his speculations, as, for example, in his hypothesis of light-quanta cannot really be held too much against him, for it is not possible to introduce really new ideas even in the most exact sciences without sometimes taking a risk.

This was written in 1913! What enabled Einstein to get through this maze was his incredible intuition. He seemed to have some kind of internal guidance system that told him what was correct—a pipe line to

the secrets of the Old One, Einstein's affectionate way of referring to God. You cannot be taught this. You have it or you do not, and the great physicists have it—no one more than Einstein in these early years. The title of the paper is interesting. In English translation, it is “Concerning a Hueristic Point of View about the Creation and Transformation of Light. “Heuristic” – “*heuristic*” in German—is such a curious choice of word. I have never seen it in the title of any other physics paper. It seems to come from the Greek. The dictionary definition is “Of or relating to a general formulation that serves to guide investigation.”⁸ Unlike relativity, he is not offering here a full-blown theory. Part of his genius was to understand that, at the time he wrote the paper, there was not, and could not be, such a theory. He was proposing some guide as to what such a theory had to explain.

The paper starts out with an introduction in which Einstein briefly summarizes the successes of the classical theory of electromagnetism—Maxwell's theory.

This theory works with continuous distributions of electromagnetic energy. It accounts for a vast domain of electromagnetic phenomena. It is the theory that Einstein discusses in his relativity paper. In that paper, written afterwards, there is only one hint that for some purposes the theory has to be modified to allow for finite numbers of “energy quanta” instead of the continuous electric and magnetic fields. In the eighth section of the relativity paper he computes the energy and frequency of what he calls a “light complex.” He notes laconically that “It is noteworthy that the energy and frequency of a light complex vary with the observer's state of motion according to the same law.” One waits in vain for the other shoe to drop. Why does he not simply say that $E = h\nu$, something that he had proposed a few months earlier? In this earlier paper he puts it very starkly,

According to the presently proposed assumption the energy in a beam of light emanating from a point source is not distributed

⁸This definition, and much more about the word, can be found at <http://www.websters-onlinedictionary.org/definition/english/he/heuristic.html>.

continuously over larger and larger volumes of space but consists of a finite number of energy quanta, localized at points in space, which move without subdividing and which are absorbed and emitted only as units.

In terms of Professor Frank's beer analogy, not only is beer bought and sold in pints and quarts, but even in a barrel you would find it localized in pints and quarts. No draft beer. It is clear how radical this is. Nothing had prepared physicists for this. The rest of the paper is an examination of "the proposed assumption."

Before doing this, Einstein has a section of the paper which he calls "Concerning Certain Difficulties in the Theory of Black-Body Radiation." In this section he answers the question that Planck should have answered. Suppose you do not know the "right answer"—Planck's distribution—but simply apply classical physics consistently to the problem, what distribution of the radiation do you find? Einstein does this and finds that the resulting distribution is neither Planck's nor Wein's. It is plotted in Figure 4.4 under the label "Classical theory." It is just the long wave length distribution that Planck had used after he had spoken with Rubens. This classical distribution which is simply proportional to the wave length cannot be right at all wave lengths. Not only does it not fit the data, but it leads to an absurdity. You can use it to compute how much radiation energy is contained in any cubic centimeter in the container. What you find is that the amount is **infinite**! This absurdity came to be called the "ultra-violet catastrophe" in honor of that fact that as λ gets smaller—goes towards the ultra-violet—the contribution to the energy at these wave lengths goes off to infinity. Classical physics, which is what is assumed to derive this distribution, has broken down.

Einstein was not the only one to have reached this conclusion. In 1900, John Strutt, better known as Lord Rayleigh, published a paper in the British journal *Philosophical Magazine* in which he presented his own radiation formula. It was an odd amalgam of the classical result and Wien formula. He offered no real explanation of how he had arrived at this formula. If Planck was aware of it he did not mention it in his papers. Einstein certainly was not aware of it. Nor was he aware of

Rayleigh's 1905 paper in which he spells out in detail his derivation of the classical distribution. This derivation is part of what we teach our students. But we have to amend it to take account of the quantum effects. Rayleigh did not bother with Planck's radiation oscillators. He went right to the radiation in the cavity. He wanted to count how many different wavelengths of the radiation could be fitted into the cavity. The restriction is that each wave has to fit inside the cavity with nothing slopping over. The answer he got was off by a numerical factor that was later discovered by James Jeans, the British astrophysicist. The resulting distribution is usually referred to as the Rayleigh-Jeans distribution. I think it might well be called the Rayleigh-Jeans-Einstein distribution. To make the final step to get it, Rayleigh used a fact from classical physics that each of these radiation "modes" in equilibrium has the same average energy. The energy is equally partitioned among these modes. This law was embedded in classical physics and led at once to the classical distribution and all the attendant difficulties.⁹ Rayleigh and Jeans both understood this but were unclear what to do about it. I do not think they saw in it the end of classical physics. In the new physics the energy is not equally distributed among the modes and this is what leads to the Planck distribution which Einstein clearly understood. He discusses the meaning of Planck's derivation in the third section of the paper.

To me, the miraculous section of this paper is the fourth which has the ponderous title, "Limiting Law of the Entropy of Monochromatic Radiation for Small Radiation Density." You can as much guess what

⁹Not to be overly mysterious about this, there was in classical physics a principle that was called the "equipartition of energy." In this instance what it implied was that each of these modes had equal average energy and that this energy was given by kT . Here T is temperature and k is the Boltzmann constant which has a value of approximately of 1.38×10^{-16} ergs per degree. The important point is that this energy did not depend on the frequency of the radiation, which is what lead to the disaster. In the Planck case the energy does depend on the frequency and is given by $\frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}$. This expression has the property that for low frequencies it goes over to the classical answer and for high frequencies it is cut off by the exponential so that the catastrophe is avoided.

this section is about from this than you can tell from the title of the painting “La Gioconda” that you are about to see the *Mona Lisa*. As I have discussed, Planck’s tactic for attempting to derive distributions was first to derive an expression for the entropy from which he could derive the distribution. Einstein turned this around. He started from the observed distribution, in particular from the Wien end where the new physics was to be found, and from it calculated the entropy. Planck had also found this expression but he had not understood what it meant. Einstein shows that it is the same expression you would find if you were considering a dilute gas of particles. The difference is that these “particles” have energies that are proportional to their frequencies; i.e., $E = h\nu$. They, are in this respect, not like the molecules of, say, a volume of hydrogen gas where the energy depends on the speeds of the particles. These “particles” all move with the speed of light. Recall from the relativity chapter, this means they are massless. Where does this leave us? The classical part of the black-body distribution is derived from the assumption that the radiation in the black body has a wave-like character. But the Wien part of the distribution is derived from the proposition that the radiation has a particle character. And in the middle? Here you have the first instance of the wave-particle duality of light. It would haunt Einstein for the rest of his life. He never came to terms with it.

In the next two sections of the paper Einstein sharpens his particle analogy and then come the last three sections of the paper in which Einstein discusses tests for this new idea. The most famous section of the entire paper is the eighth. It has the cumbersome title “On the Production of Cathode Rays by Irradiation of Solid Bodies.” To explain it a little history is in order. In 1886, the German physicist Heinrich Hertz performed the experiments that successfully showed that Maxwell’s prediction that light was an electro-magnetic wave was right. In the course of this he found that a spark he was using to detect the radiation was enhanced if ultra-violet light struck the metal of his detector. He had no explanation for this but, being a good experimenter, noted it. As it turned out, in the same set of experiments he had, unknown to him, detected both the particle and wave properties of light. But that realization

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would only come later. In 1897, the British physicist J.J. Thomson identified the electron as the particle that was emitted from the cathode—the negatively charged component of a vacuum tube. These electrons became known as “cathode rays.” In 1899, he showed that these same cathode rays were emitted when ultraviolet light was shone on a metal. What Hertz had actually seen were these electrons. The next important step was taken just after the turn of the century by the German physicist Philipp Lenard. Lenard had a carbon arc light whose intensity he could vary by a factor of a thousand. This light was shone on a metal plate and the emitted electrons collected on a detector plate. He was able to measure the energy of the electrons. The result was totally unexpected. Common sense, and indeed classical physics, would have predicted that when the intensity of the light was increased the emitted electrons would have a higher energy. But this is not what happened. In general, the electrons are emitted with various energies but one can focus on the maximum energy. What Lenard found was that this energy did not change when the intensity was increased. What happened was that number of emitted electrons carrying this energy increased. He then very cleverly broke up the light from his carbon arc into various frequencies using a spectrometer. He seemed to find that the higher frequency components of the light caused electrons of higher energies to be emitted, but the data was not conclusive.

In this section of Einstein’s paper he explains everything in one equation that is so simple that you could teach it to high school students. It was this equation for which Einstein was awarded the Nobel Prize for 1921, which he collected in 1922. The citation of the Royal Swedish Academy that awards these prizes is quite marvelous. It reads,

ROYAL SWEDISH ACADEMY has at the assembly held on November 9, 1922, in accordance with the stipulation in the will and testament of Alfred Nobel dated November 27, 1895, decided to/independent of the value that/(after eventual confirmation) may be credited to the relativity and gravitation theory/bestow the prize/that for 1921 is awarded to the person within the field of physics who has made the most important discovery or invention/

to Albert Einstein being most highly deserving in the field of theoretical physics/particularly his discovery of the law pertaining to the photoelectric effect.¹⁰

The “photoelectric” effect is the shorthand name for the phenomenon of the emission of electrons by light. Incidentally, the Prize was worth about 32,000, 1922 dollars, all of which went to his ex-wife as part of a divorce settlement that had been made several years earlier. What then is the equation? I will not use the notation of Einstein’s paper since he has a way of rendering Planck’s constant “ h ,” which is a consequence of how he defines the Wien distribution, that is rather ungainly. In modern notation the equation is simply

$$E_{\max} = h\nu - P.$$

Here E_{\max} is the maximum energy an electron can have when it is liberated by a quantum that has energy $h\nu$. The reason that it cannot have all the energy is that some is needed to allow the electron the escape from its binding to the metal surface on which the light is incident. This is what “ P ” stands for. This equation is simply the conservation of energy. It says that if N quanta of energy $h\nu$ are incident on the surface then N electrons with a maximum energy of E_{\max} can be released. It furthermore says that the energy of the electrons increases with the frequency of the light. This explains Lenard’s results. The first sentence of this section of Einstein’s paper reads, “The traditional view that the energy of light is distributed continuously through the region illuminated by the light runs into great difficulty in trying to explain photoelectric phenomena, as was outlined in a trail-blazing paper by Lenard.” Two decades later Lenard had become a rabid anti-Semitic Nazi and all of this was, according to him, “decadent Jewish physics.”

In the next, and final section, “Afterward,” I will describe the experiments that confirmed this equation and end with a brief overview of Einstein’s attitude towards the quantum theory as the theory evolved.

¹⁰I am grateful to Gerald Holton for supplying the translation of the Swedish original.

➤ AFTERWARD

I cannot make a case for my attitude in physics which you would consider at all reasonable. I admit, of course, that there is a considerable amount of validity in the statistical approach which you were first to recognize clearly as necessary given the framework of the existing formalism. I cannot seriously believe in it because the theory cannot be reconciled with the idea that physics should represent a reality in time and space, free from spooky actions at a distance.

—Einstein to Max Born, 3 March 1947

The definitive experiments on the photoelectric effect were carried out by the American physicist Robert Millikan in the years from 1914 to 1916. They confirmed Einstein's equation and for them Millikan won the Nobel Prize in 1923, two years after Einstein. One might think that Millikan's experiments would have settled the matter in the sense that the quantum would have achieved universal acceptance. This was far from the case and Millikan is a prime example. In 1917, Millikan published a book called *The Electron*, in which he describes these experiments. He writes,

Despite then the apparently complete success of the Einstein equation, the physical theory of which it was designed to be the symbolic expression, is found so untenable that Einstein himself, I believe, no longer holds to it, and we are in the position of having built a very perfect structure and then knocked out entirely the underpinning without causing the building to fall. It stands complete and apparently well tested but without any visible means of support, and the most fascinating problem of modern physics is to find them. Experiment has outrun theory, or, better, guided by erroneous theory, it has discovered relationships which seem to be of the greatest interest and importance, but the reasons for them are as yet not at all understood.

Millikan was correct in the sense that the quantum theory, which was barely in its infancy, was not well understood. But his notion that

Einstein had abandoned the quantum as an explanation of the photoelectric effect is absurd. Indeed, typically, after having published his explanation of the photoelectric effect Einstein found another application of the quantum idea. This had to do with applying Planck's oscillator model with individually quantized oscillators to the study of the absorption of heat in solids. The history of this subject goes back to 1819, when two young French physicists, Pierre Louis Dulong and Alexis Thérèse Petit studied the heat absorption of a variety of materials—mostly metals. They found that, taking a standard amount of the materiel, the amount of heat needed to raise the temperature by, say, one degree, appeared to be the same for all these elements and did not depend on the temperature of the material. This law was readily derived from a model involving classical oscillators. The problem was that by the turn of century it seemed to be breaking down at the lower temperatures. In 1906, Einstein supposed that in reality the energies of the oscillators were "quantized." Only integer multiples of the basic energy unit was allowed. Using this assumption, he found a new law for the specific heat which agreed with the old one at the higher temperatures and with the experimental data at the lower ones. This study was the first to apply the quantum theory to solids. It persuaded some people who had not taken the quantum seriously before to take it seriously now. The next great step was taken in 1913, by Niels Bohr. What Bohr did was to quantize the electron orbits in atoms. The electrons were only allowed to travel in restricted orbits. When an electron "jumped" from a higher to a lower orbit it emitted a quantum of radiation equal to the energy difference between the two orbits. This incidentally explained the results of Bunsen and Kirchhoff on the dark sodium spectral lines. Absorption of light was subject to the same quantum rules. For light to be absorbed it had to have the right energy to induce of these quantum jump. Sodium light could be absorbed by sodium vapor because it had the right energy. Einstein was extremely enthusiastic about Bohr's work.

In 1916, Einstein presented a new way of looking at Planck's distribution. It cannot exactly be described as a derivation because there was

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as yet no theory from which to derive it. But one of the basic ideas became decades later the basis of the laser. The idea was to consider a model in which, say, an electron, could be in one of two energy states. There was a state of least energy, the so-called “ground state” and an excited state of greater energy. This system was assumed to be in a bath of radiation at some temperature. Two obvious things can happen. If an electron is in the ground state it can absorb a radiation quantum and jump to the excited state. Once in the excited state it can spontaneously emit a quantum of radiation and jump back to the ground state. But Einstein discovered that there was a third process. The radiation bath can stimulate the excited electron to emit a quantum. It was this stimulated emission that was exploited in the laser since it led to a method of amplifying the intensity of the emitted radiation.

In 1923, the French theoretical physicist Louis de Broglie in a proposed PhD thesis, suggested that particles like electrons might also have a wave character. This completed the wave-particle duality. Light was both a wave and a particle, and an electron was both a particle and a wave. De Broglie’s thesis advisor, Paul Langevin, sent a copy of the thesis to Einstein, who replied that he found the ideas interesting. The wave nature of the electron was demonstrated experimentally four years later. At first, it was assumed that these were waves like light waves in that they oscillated in ordinary space. They might act as “guides” for the particles. Indeed, in 1926, the Austrian physicist Erwin Schrödinger found the equation that bears his name which described these waves. Einstein was delighted with this until Max Born, and others, showed that these waves were different. In fact they were waves of probability. Where they had large amplitudes, a particle was most likely to be found. It was at this point that Einstein and the quantum theory parted company. Indeed, in 1926, in a famous and often quoted letter to Born he wrote,

Quantum mechanics is certainly imposing. But an inner voice tells me that it is not yet the real thing. The theory says a lot, but it does not really bring us any closer to the secrets of the “Old One.” I, at any rate, am convinced that *He* is not playing dice.

Secrets of the Old One

Einstein then entered a period in which he tried to show that the theory was wrong. This inspired monumental debates with Bohr which, in truth, Einstein lost, but we all won since they clarified the theory. Finally, Einstein decided that the theory was not a complete description of “reality.” Usually when physicists begin talking about reality it is a bad sign. He tried for the rest of his life to produce a theory that would replace quantum theory. But he was doing this without any guidance from experiment so there was little chance for success. Many people tried to induce him back into the mainstream. John Wheeler told me that when Feynman, who was his student, produced a new formalism for the quantum theory Wheeler thought that it was so beautiful that Einstein would surely be converted. Of course he was not. I think that Einstein was in his heart a classical physicist. Poets, as Cocteau once said, tend to sing from their family trees. Einstein’s family tree was classical physics—the physics he learned as a student—and while quantum theory was, in a sense, his child, it was not a child that he was able to adopt.

Our presentation of the four essays is now done. In the brief epilogue I will trace the arc of the remainder of Einstein’s life.

